

MULTICOMMENTED TRANSCRIPTS METHODOLOGY AS AN EDUCATIONAL TOOL FOR TEACHERS INVOLVED IN CONSTRUCTIVE DIDACTICAL PROJECTS IN EARLY ALGEBRA

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This paper presents results regarding the Multicommented Transcripts Methodology (MTM) we have enacted to promote in teachers of primary school and secondary school awareness of their own ways of being in the class and to guide them in managing mathematical discussions. After a brief overview of the theoretical framework and the methodological structure of MTM, two multicommented classroom-based episodes are proposed, with the aim of highlighting the MTM educational potentialities. Some final remarks concerning the formative, cultural, educational and methodological principles of MTM are made.

Early algebra, Metacognition, Multicommented Transcripts Methodology,
Reflective Teaching, Teacher Education

INTRODUCTION

Early algebra is proving to be an appropriate approach to algebra for 5 to 14 years-old pupils, which allows them to achieve a better control over the meaning of the algebraic objects as well as of their generative processes. This achievement may occur in different ways depending on the age of exposure to early algebra, either in the first years of primary school or after several years of traditional teaching. In the first case, arithmetic should be *introduced* in a *pre-algebraic* perspective, whereas in the second one it should be *revisited from a relational and structural point of view* overcoming the traditional focus on algorithms execution. This entails a re-framing of teaching in the arithmetic-algebraic area requiring a greater attention to the construction of algebraic language as an instrument for representing relations and properties. This change of perspective leads teachers to revise their own knowledge, beliefs, attitudes, working styles.

The MTM, on which we report here, was born for this aim. It developed within our ArAl Project, which involves in-service teachers in long term educational process. In the project the teachers deal with basic theoretical issues in early algebra together with the development of teaching sequences across the school grades: from algebraic generational activities to meta level activities (Kieran 1996), such as modeling and proof¹. This led us to design ways and tools to study the behavior of teachers involved in our project and engaged in early algebra teaching sequences, with the aim to lead them to

reflect on their actions in the classroom and understand how these may be improved. (Malara & Navarra 2009, Cusi & Al. and related references).

SOME THEORETICAL INDICATIONS

Several studies highlight how teacher's knowledge, beliefs, emotions and attitudes are intertwined and determinant components of teaching and learning processes (see Malara & Zan 2008 and related references). They underline that the study of these relationships is crucial to provide teachers with useful suggestions in their professional development. In this respect, the analysis of interactive and discursive practices, the awareness of the variables that influence the classroom process and self-observation during action are fundamental. The value of the teacher's *critical reflection* is a well known fact in the achievement and empowerment of the above mentioned skills (see for example Mason 2002, Jaworski 2003). In Jaworski's view, the essence of the reflective practice consists of making explicit teaching approaches and processes, so that they become the object of a detailed critical examination. She promotes the usefulness of *communities of inquiry in teaching*, discussion groups composed of teachers and researchers, in which the teacher has the opportunity to develop a specific identity.

Our teacher training model follows these conceptions and modalities. But it represents the outcome of research and training practices developed in Italy since the 1970's. Our hypothesis is that by critically reflecting on socio-constructive teaching/learning processes, the teacher is led to become aware of the different roles he/she is supposed to play in the classroom, of the best modalities to interpret them and can also get useful suggestions about how to behave in the classroom. Moreover, it is crucial for teachers to approach research results that can be useful for practice and become aware of the importance of studying them for their own professional development.

The focus of our research is on the *analysis of classroom-based processes* that develop along teaching sequences planned with the teacher. These studies aim at: showing teachers the *micro-situations* which compose a process and the higher or lower effectiveness of the *micro-decisions* made; favoring the achievement of control over their own behavior and communication styles, as well as noticing the impact the latter have on pupils' behavior and learning. More in general they aim at gathering both theoretical and practical tools for pre-service and distance teacher training.

THE METHODOLOGY

In our project teachers are involved for at least two years in training activities. After planning and implementing lesson units together with the

researchers, the teachers carefully record some lessons they choose, transcribe them according to a predefined format, add details coming from the notes they have taken during the lessons (gestures, expressions, etc.) and include reflections and comments. The teachers then engage themselves in a network exchange of e-mails with their mentors and sometimes with other teachers. The exchange consists mainly of reflections and comments regarding the classroom transcripts, through which the mentor infers the teachers' interpretation of their theoretical frame and the developed cultural values, as well as their progressive harmonization with the background and the previous attitudes. This is the core of MTM.

There are two different ways of implementing MTM. One way takes place in a university environment as part of a national or international research and training programs. It involves a small number of researchers and teachers who strictly interact with specific research proposals. The other way is implemented in schools, from different Italian regions and organized in networks, by teachers who take part into the ArAl Project. It is characterized by a few meetings with researchers and teachers and several long distance interactions, conducted via e-mail. A high number of teachers are involved (in the year 2010 nearly 150), organized in small groups of work. Each group is coordinated by a researcher, who plays the fundamental role of E-tutor. This latter type of intervention is mainly aimed at training although with important spin-offs for research.

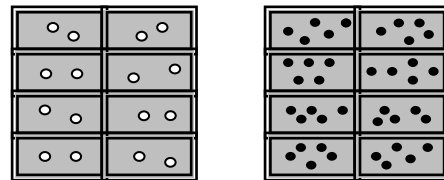
As first step, teachers are required of including in their transcripts of class session, either positive or negative comments concerning mathematical issues or critical points in the development of the discussion, possibly attaching some class material. The transcript of each session is sent by e-mail to the E-tutor, who makes comments and sends it to other teachers and researchers involved for further comments. Each of them can intervene again in the cycle with further meta-comments. So, the multicommented transcripts (MT) reify. They become an important object of study for the teachers through sharing with colleagues within the school and during meetings with the E-tutors. They are also published in the schools websites, in some cases included in www.aralweb.unimore.it as 'good practices'. In the following we wish to highlight their educational potentialities through some excerpts.

ANALYSIS OF CLASSROOM EPISODES: TRANSCRIPTS AND COMMENTS

Here, two MT excerpts, which document both the interactions among the actors and the variety of the faced themes, are presented. The order of their presentation is: (a) context; (b) transcript of session; (c) comments. In the comments, the words written in *Italic* indicate key elements of the ArAl Project theoretical framework, which are described in its Glossary ² (some examples can be seen in <http://www.aralweb.unimore.it/online/Home/ArAlProject/Glossary.html>).

Episode 1 (year 3 primary, 8-9 years old)

The teacher is participating in the project for the second year and she is working on the distributive law, already discovered by the class in simpler cases.



She shows two boxes, divided in eight parts, containing two types of marbles, as shown above. She says they belong to Marina's collection who has organized them in a very orderly way. Then the teacher formulates the task: "Represent the situation in mathematical language so that Brioshi ³ may find the total number of marbles in the two boxes". Pupils work and their proposals are written on the blackboard. Many of them formulate more than one proposal.

Andreina, Danilo, Francesca, Martina	$16 \times 40 = n$ $40 \times 16 = n$ $n = 16 \times 40$ $n = 40 \times 16$
Andrea	$(5 \times 2) + (4 \times 2) = n$ $n = (5 \times 2) + (4 \times 2)$
Maria	$2 \times 8 = n$ $5 \times 8 = n$ $n = 2 \times 8 + 5 \times 8$
Bruno	$(2 \times 8) + (5 \times 8) = n$
Melania	$4 \times 2 + 4 \times 5$
Sara, Elena, Giovanna	$5 \times 8 = n$ $2 \times 8 = n$ $n = 5 \times 8$ $n = 2 \times 8$
Francesco	$(2 \times 8) + (5 \times 8) = n$
Chiara	$2 \times 2 + 5 \times 2 = n$ $n = (2 \times 8) + (5 \times 8)$ $n = 4 + 10$

- Teacher: Good! Now, as usual, let's open up the discussion. [Comm 1]
- Andreina: Teacher, we were wrong because 16 is not repeated 40 times.
- Teacher: Explain it better.
- Andreina: I understood that we didn't have to multiply red marbles and green marbles, but rather put them together.

Teacher: What do you mean by ‘put together’, try to use mathematical language better.

Andreina: United...

Teacher: Do you know a more suitable term to explain what Andreina means?

Francesco: You must add.

Teacher: Yes, this sounds clearer... Any other remark?

Melania: In my opinion the translations made by Andreina’s group are *opaque*.

Teacher: What do you mean?

Bruno: They are *opaque* because they have already found the number of marbles.

Chiara: It was not up to us to find 16 and 40, but rather write the translation to be sent to Brioshi. They have nearly solved the problem.

Bruno: It’s true, they found the *product and not the process*.

Teacher: What do you think about Andrea’s representation?

Andrea: Miss, I got wrong too... erase, erase.

Teacher: Hold on, explain what you wrote (*Andrea can’t explain*).

Melania: I also realize that I forgot to write something. I wrote 4×2 and 4×5 because I saw separate columns, but now I understood that my representation is not complete, I must add ‘ $\times 2$ ’.

Teacher: Tell me which changes I should make.

Melania: $4 \times 2 \times 2 + 4 \times 5 \times 2 = n$.

Francesco: But she wrote like me... like Bruno... and like Maria Giovanna, because 4×2 equals 8.

Andrea: Melania factorized eight!

The expression by Francesco is chosen to be sent out to Brioshi: $n = 2 \times 8 + 5 \times 8$.

[Comm 2] [Comm 3]

1. Comment by the E-tutor: Regardless how correct the expressions are, the pupils show they use the *letter* as indicator of a number to be found. It is a naive and sometimes not pure use of the letter, like in the cases of Maria, Sara,

Giovanna, Elena and Chiara, where the same letter stands for different quantities. Pupils should be led to reflect upon this from the beginning.

2. Comment by the E-tutor: The transcript shows how the class is familiar with *mathematical discussion*. Also, it shows the good argumentative skills of the pupils and the fact that they draw on important theoretical constructs such as the distinction between *opaque* and *transparent* representations and between *process* and *product* of a calculation. It would be appropriate not to overlook a collective investigation on expressions like Melania's, which are incorrect but revealing her initial vision of the situation. How do they get to decide that Francesco's expression should be sent out to Brioshi?

3. Comment by the coordinator: The iconic representation proposed by the teacher for translation into mathematical language is problematic and deserves reflection. Its negative influences can be detected in the translations made by Andrea or Melania. Many pupils use the (correct) representations 2×8 e 5×8 . But the numerical representation consistent with the given representation is: $2 \times 4 + 2 \times 4 + 5 \times 4 + 5 \times 4$. Moreover, reading by rows, one may be led to the representation $(2+2+5+5) \times 4$, changeable into $(2 \times 2 + 5 \times 2) \times 4$, for the meaning of multiplication as repeated addition, an expression which can be, in turn, modified into $(2+5) \times 2 \times 4$, for the distributive law they just met. The latter expression permits a link with representations 'perceived' by many pupils: $(2+5) \times 8$ e $2 \times 8 + 5 \times 8$. These steps are certainly very sophisticated for pupils aged 8-9 and require control over parentheses and the property itself. The teacher should appropriately make hypotheses about the possible interpretations induced by an iconic representation and constructs a discussion sketch for each of them, in case some pupils propose it or even to show how a representation can be viewed in more different ways. A general point must be highlighted: the need to favor the interpretation of paraphrases in mathematical language contributing to the construction of meaningful skills in pupils.

Episode 2 (grade 9, pupils aged 11-12)

A general representation of the sequence 4, 11, 18, 25, 32, ... is sought for. The table reported below is constructed on the blackboard to identify the link between place number and term of the sequence.

Place [Comm 4]	Term	Rule
1	4	$7 \times 2 - 10$
2	11	$7 \times 3 - 10$
3	18	$7 \times 4 - 10$
...
8	53	?

Pupils find various rules, but they are focused on one in this case. The teacher is encouraging reflection upon relationships between numbers of the first and the third columns.

Teacher: You have discovered that “the place equals the term preceding the second factor of the multiplication”. So, for instance, what is the rule at place 8? [Comm 5]

Serena: You must do $7 \times 9 - 10$.

Teacher: Do we all agree? [Comm 6]

Many: Yes.

Teacher: And is it 53? [Comm 7A] [Comm 7B]

Many: Yes.

4. Comment by the E-tutor: ‘place’ is used instead of ‘place number’. It is appropriate to be precise not to induce pupils to identify quality and quantity.

5 Comment by the E-tutor: Pay attention to the abbreviations (see Comm 4). The fact that pupils know the meaning of ‘preceding’ in Italian is not enough for a *translation* in algebraic language. They must learn to express ‘preceding’ in relation to the number that follows. If they are able to paraphrase it only with ‘that precedes’ (as in the transcript), they get stuck, because these *paraphrases* are *opaque*. They should rather be induced to make explicit the link between the two numbers and express the preceding number as a function of the subsequent. Natural language supports the achievement of this expression. In a grade 8 class, for instance, the proposal made by a pupil: ‘the preceding number is always one unit smaller than the number that follows’ turned out to be very effective and decisive for translation.

6. Comment by the E-tutor: There should be a *shared didactical contract* according to which monosyllabic answers are not acceptable. They do not help the teacher understand how the topic has actually been understood and do not help classmates either. Questions that require only ‘Yes’ or ‘No’ answers are not productive as well. Pupils do not *argue*, they only answer the teacher’s questions and let her guide them towards her objective. Requests like ‘Explain what you mean’ make the pupil himself define the objective of his reasoning, and construct the related explanation. In these cases, the teacher should only evaluate the quality of the discussion, *sorting out the argumentative traffic* by intervening mainly on the methodological plane, thus favoring the *social construction of knowledge* through *negotiation, sharing* and *stabilization* of meanings. A teacher should become aware that control over natural *language*

and implementation of *social practices* are basic elements for the understanding of mathematical language and therefore of mathematical concepts.

7A. Comment by the teacher: At this point (I refer to Serena's 'You must do' and to my 'it is', but this also occurs elsewhere in the transcript), I realize how imprecise my language is. I could have said "Does $7 \times 9 - 10$ 'represent' number 53?" or "Does it 'correspond' to 53?"

7B. Comment linked to the previous one by the E-tutor: Ok, right. But it is not only a matter of language, I think this reveals rooted attitudes which reflect hidden convictions. Very often teachers' algorithmic approach is 'dominant' (referring to: operations, result, calculate, solve, 'how much is', 'it is', ...), the relational one is 'recessive' (mainly focusing on: *relations, structure, representation, ...*). These activities in an early algebra environment aim to induce teachers to reflect on this point.

REFLECTION ON EPISODES AND COMMENTS

Comments in the transcripts are valuable for training in several aspects. Some of the most meaningful are reported below.

Socio-linguistic aspects. We underlined how important linguistic aspects are in the construction of mathematical knowledge and how central they are in mathematical discussions. Pre-requisite for teachers to be able to make the discussion a shared instrument for the class is that they acquire many skills: to create a good context for interaction, to enact socio-mathematical norms that lead to compare different solutions, evaluate if a solution is acceptable or of a good quality, to steer the direction of the discussion in the different phases, to involve pupils in meta cognitive acts and so on.

The relationship theory-practice. Another aspect emerging from comments is reference to the theoretical framework and to the glossary of the project not only shared by researchers and teachers but also – with appropriate adjustments - by teachers and pupils. Sharing is extremely important in both cases, because teachers and above all pupils are enabled to understand how aspects apparently far from mathematics, such as: 1) competence in using languages, mainly natural language, and control of their semantics and syntax; 2) being able to translate from one language to another; 3) difference between representing and solving a problem situation; 4) distinction between process and product; 5) recognizing the meanings of the equal sign; ... *are the foundations of a meaningful construction of mathematical knowledge.*

Mathematical aspects: an example, the conquest of the letter. In the first episode the letter is used in a very naïve way, in the second one it represents

a high-level goal (in the next step the variable 'n' is introduced as place number and to represent $7 \times (n-1) - 10$ by t_n as the n-th term of the sequence). *Algebraic babbling* (a theoretical constructs of the ArAl project, which compares modalities of construction of algebraic language to those of construction of natural language) emerges throughout exploration, discovery, conjectures, failed attempts which entail the introduction and use of the letter with various meanings (generic number, unknown, variable). Through transcripts and the analysis of comments, teachers become aware that the main difficulty for pupils is *to get to understand that a letter can represent a number*. It is an epistemological jump, fundamental for algebraic thinking, which may become a block if the pupil is not guided enough.

The use of comments in the training process aims to make the teacher sensitive to basic general issues, such as: are students aware they are *communicating* through mathematical language? What kind of relationship do they have with the *semantics* and *syntax* of mathematical language? Which environment (situation, contest) can improve algebraic thinking? How can one detect the awareness of 'algebraic content' in pupils' sentences, intuitions, proposals, representations? These kinds of questions make teachers' reflections profound, meaningful and productive.

CONCLUDING REMARKS

MTM appears to be an effective instrument in teacher's training processes involving mathematics.

This methodology has an important pre-requisite: *a trusting relationship between teachers and researchers*. Moreover, when the teacher edits a transcript, he puts himself on a different level. He detaches himself from the activity he was part of and critically reads what happened in the classroom. His class is no longer his class. His transcript is no longer narrative, it acquires scientific aspect and becomes a training instrument. Comments may bring her/his misconceptions to the surface, touching sensitive points. Many teachers immediately realize that the comments are valuable and accept the remarks. Others see their competence jeopardized; they feel uncomfortable and refuse to accept that their transcripts may become public. Others tend to 'watch and wait', they need time to familiarize themselves with the methodology and be convinced before using it. These different types of behavioral patterns are monitored by researchers, who are always trying to make teachers understand that MTM is meaningful and productive only if participants engage in the project with open and sincerely committed minds.

Notes

1. For an overview of the project and related bibliography, see the site www.aralweb.unimore.it.
2. Glossary terms are more than 100 and belong to several categories: theoretical constructs, which are both original or coming from previous studies of mathematics education, terms relating to linguistic or psychological aspects. These are interconnected in a network of references, which allows the teacher to build a reticulum of knowledge that led him/her gradually to a new vision of the arithmetical-algebraic area and its teaching.
3. Brioshi is a metaphor from the ArAl Project. He is a virtual Japanese student exchanging messages in mathematical language with pupils. His acknowledged skill in this area, encourages pupils to check the correctness of the mathematical expressions to be sent out to him.

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