

On some aspects of the assessment of pupils involved in the solution of non standard metacognitive word problems

Giancarlo Navarra, Department of Mathematics, University of Modena, Italy
tel 36 59 364677 - fax 36 59 370513 - E mail: MALARA@UNIMO.IT
Private address: via Cugnach 4, 32036 Sedico (BL), Italia - tel/fax 36 437 296978

***Abstract.** The paper presents the first results of a research carried out with 80 pupils aged 8 to 10 who were requested to help a hypothetical classmate having difficulties in solving some word problems where one or two pieces of information were missing. The about 370 responses were evaluated both from the metacognitive point of view (the quality of help) and from the cognitive one (the form of the solution of the problems) and allowed to organize a system of formative assessment structured in nine voices useful for teachers evaluating pupils involved in non-routine word problems.*

1. For several years our research group has been dealing with methodological and curricular innovation in primary school (pupils aged 7 to 10) and in lower secondary school (pupils aged 11 to 14) and has faced among other things the didactics of problems and the assessment of the competence of pupils engaged in problem solving activity.

To set up innovative didactics on problem solving in Italian School means to promote activities which influence the widespread conception of problem as an application exercise, giving it a more significant role in the construction of curricula. For this reason it is necessary to attend to the choice of the problematic situations, to the way of working of the teachers and to the assessment of the pupils engaged in tasks differing from traditional ones.

2. In the research, focused on word problems, we concentrate our attention mainly on linguistic competence, since we firmly believe that it is one of the main causes of success or failure in problem solving too. For this reason the pupils who participate to the activity (about 80 aged 8 to 10) were asked to motivate their written answers by taking a big care of their communicability as well as of their mathematical correctness. Verbal communication and collective discussion about the pupils' protocols analyzed and compared with the overhead projector were very important, above all from a metacognitive point of view.

The precise analysis of 370 protocols, attentive to the ways in which the pupils organized their answers, so as to point out cognitive styles, difficulties, misconceptions and so on, is an important methodological aspect.

3. At the beginning we proposed to the pupils this situation (which presented important and motivating social-affective implications during all the activity):

A schoolmate, Piera, has tried to solve some problems, but she couldn't do it and asks you to help her.
Could you explain her why she is unsuccessful?

After this introduction we gave a sequence of nine problems where one or two pieces of information were missing. Here are some examples:

(P1) Andrea bought a magazine. The shopkeeper gave him 350 lire change. How much was the magazine?
(P2) A staircase is 20 m high from bottom to top. How tall is each step?
(P3) Our school has two school-buses. Every day one of them covers 12 km.

How many kilometers do school-buses cover from monday to saturday?
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We opportunely filed the problems on the basis of their difficulties and of the nature of the involved informations (explicit, implicit, absent). We present a scheme of the filing:

P	explicit informations	implicit informations	missing informations	questions
(P1)	change: 350L	—	money given to the shopkeeper	cost of the magazine
(P2)	height of the staircase: 20 m	<i>each step is the same height</i>	number of the steps	height of one step
(P3)	number school-buses: 2 kilometers a day: 12	<i>days from monday to saturday: 6</i>	km a day covered by the other bus	kilometers covered by two buses

The assignment did not explicitly request to solve the problems but set the pupils free to get organized as they thought best. It is a fact that many of them thought it better to insert in the help to Piera the solution (variously argued and correct) as the complexity of the text grew.

4. The protocols were classified from two different points of view everyone of which involved the attaining of some aims:

- (i) the metacognitive one (does the pupil clearly observe the assignment to help Piera?);
- (ii) the resolutive one (is the pupil able to analyze the problem? To elaborate the solution? To use his knowledge?)

We used the classification to organize a system of formative assessment structured into nine voices which is at the moment in course of definition and of testing in the same classes which keep on working with this kind of word problems.

I Texts showing the general attaining of the aims (**B** and **C** can be placed in the Vygotskian zone of proximal development). The pupil

A understands both the problem and the assignment. He/she shows a good control both on cognitive and on metacognitive level.

B basically understands the logic structure of the problem but he/she has a partial metacognitive control over the situation: he/she offers an argued solution (inventing the missing information) but he/she does not help to understand why someone is not able to find it.

C understands the problem but has a poor metacognitive control over the situation: he/she merely suggests the operation/s (inventing the missing information).

II Texts showing an only partial attaining of the aims. The pupil

D gives the numerical result (often incorrect) only and does not justified it. The metacognitive control is absent.

E has the metacognitive control but gives an incorrect explanation because he/she misinterprets the structure of the problem confusing the missing information with the question or with one of the data.

F expresses an embryonic comprehension of the logic structure of the problem. He/she merely suggests the operations, normally without explaining them (if there were words, they are edging ones).

III Texts showing an inadequate attaining of the aims. The pupil

G has a merely outside metacognitive control organized on formal problem solving aspects only (one does not write the data, does not draw a sketch, and so on).

H has a metacognitive control limited to aspects having nothing to do with mathematics (inattention, tiredness, excessive difficulties and so on) at most accompanied by the

verbal exposition of a sequence of incorrect operations and/or by a not motivated result (generally wrong).

I shows a generalized lack of understanding (for example: “I do not know how to help you”).

5. Being the work in progress, the system of assessment is still in phase of formulation and of sharpening; it has the aim of planning a tool of evaluation for teachers involved in non-routine word problems.

In the course of our speech we:

- will illustrate the classification of the protocols on the basis of which we are elaborating the system of assessment;
- will present many examples of the protocols;
- will analyze the distribution of the voices of the classification relating to the age of the pupils, the difficulties of the problems, and so on;
- will investigate the system of assessment.

References

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