EXPLORATIVE ITINERARIES IN THE POP-UP GEOMETRY

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We describe an experimental activity on the use of *pop-up paper engineering* as an instrument for promoting a dynamic switching from plan geometry to space geometry and vice versa. The pupils learn to represent on the plan the cuts and the folds which enable them to obtain pop-up solids. To do it, they learn to imagine threedimensionally the objects <u>before</u> realizing them. The age of the pupils and the aims of the teachers lead to favour expressive or mathematical aspects. The realization of a book represents the goal for the activity. This paper is linked to an exhibition containing detailed experimental itineraries and examples of pop-up books planned and made by children.

Pop-up books

Pop-up books are (or should be) in every children's library.

Action book, livre animé, libro animato. It was not until the 18th cenury that these books were producted specifically to amuse children. They ingeniously incorporated flaps to turn, peep-holes and cut-outs. The first true pop-up was *Little Red Riding Hood* and was published in 1855 in London. Gradually German publishers came to dominate the rapidly developing techniques of colour printing and book production.

In 1925 Josef Albers and Laszlo Moholy-Nagy in the Bauhaus began to attend to mechanisms by which papers can be transformed into tridimensional objects.

Today pop-up books are still planned essentially *to amuse*, but in the last years these techniques are used to produce pop-up books with serious educational intents.

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Pop-up paper engineering

The *pop-up paper engineering* is the set of techniques based on paper cutting and folding which enables to realize mechanisms through which the figures of books, cards, dioramas burst into three-dimensional shapes when they are opened. There are many techniques according to the geometric aims: movement of parallel plans, rotations, slipping plans, and so on.

Every time a *process* of *transformation* takes place. The pop-up object is drawn on a paper and than it becomes *tridimensional* through appropriate cuts and foldings. Between the two phases, there is the *movement*. The result is extraordinary different from the starting-point not only because of the third dimension but also because it is obtained in a *dynamic* way. And it is surprising because it is sudden and unexpected.

The *imagination* is necessary to link the two phases (planning - carrying out) and to support the conscious anticipation of the outcome of the transformation. One must learn *to see* on the paper something which at that moment exists only *in his mind.*

Errors are inevitable but generally they are evident and the correction is handy. So, even the youngest pupils realize that what they have obtained is not what they had in their mind and spontaneously experiment longer to attain their aims.

A dynamic geometry

A pop-up is realized in three phases: (i) a project is made on a sheet of paper or cardboard, (ii) the cut lines and the folds are individuated; (iii) the sheet is cutted and folded.

It is possible to check and to correct processes during the realization of a pop-up but, if the planning is wrong, it is often necessary to begin all over again. So, the main gift of a pop-up planner *is to imagine three-dimensionally the object before realizing it*, which is to foresee future cuts and folds.in the lines of the project.

Empirical experience, consideration on mistakes and consequent finding out of rules count for much in this process. One must anticipate what parts of the sheet will become horizontal planes and what vertical ones, what will be foreground and what background, what upper and what lower, what high and what large, and so on *before* realizing the pop-up

From plan geometry to space geometry and vice versa

Hence the didactic potentiality of pop-up: it can be a vehicle for a *dynamic geometry* through planning, carrying out, manipulating and opportunely aided conceptualizing activities. From a general point of view, pop-up can support *the passage from plan geometry to space geometry* and vice versa.

According to the geometric principles taken into account, every pop-up mechanism can be analized at increasing levels of complexity. The mechanism explored by pupils aged 6-10 enables to realize *working plans which move in parallel with the base plans out of which they have been obtained by cutting and folding. Cuts are parallel one another as well as the folds. Cuts and folds are perpendicular one another.*

It is just the exploration which can stimulate a didactic use of pop-up: to *investigate* the mechanisms, to *foresee* the results of cutting and folding and to *verify* operatively the surmises, to learn by the mistakes and from them to derive new curiosities.

Naturally the age of pupils and the didactic aims of the teachers lead to favour *expressive* or *mathematical* aspects. It is natural *to privilege the first ones as a bridge to* *the second ones* above all with youngest pupils (we will come again to this subject in the conclusions).

Some pop-up solids: the cube, the "prolonged cube", the cuboid

fig.1a

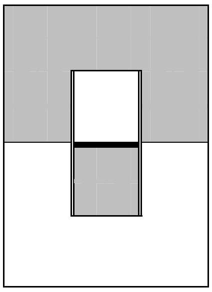


fig.1b

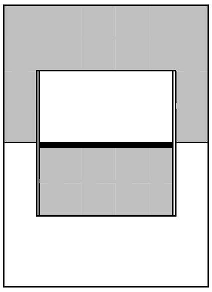


fig.1c

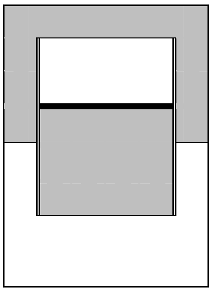


fig.2

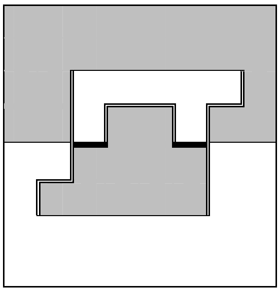
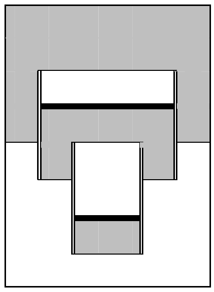


fig.3



Children aged 5-7 explore and realize pop-up objects based on three 'solids': the cube (fig.1a), its 'derived' 'prolonged cube' (so named by pupils, fig.1b) and the cuboid (fig.1c).

These solids become easily the starting-point for successive explorations during which one discovers that cuts and folds can be modified (fig.2) and that solids composed by variously superposed parallelepipeds can be built (fig.3). The unlimited expressive chances of pop-up come out of these discoveries.

Infant school (pupils 5-6 years old) – primary school (level 1, pupils 6-7 years old)

We make pupils discover pop-up universe by means of classic stories (Little Red Riding Hood) or original tales (Celestino in Flatland, inspired to Adwin Abbott's novel) arranged through animated events in form of 'pop-up theatre'. Then they begin to plan and to realize pop-up objects (houses, furnishings, trees, bushes and so on) exploring cube (fig.1a) and its possible variations (fig.2). During these phases they meet simple geometrical and topological concepts (mainly inside-outside and over-under) and improve their comprehension of relevant difference between *hill fold* and *valley fold*.

Later they build simple pop-up books. It is evident that language, logic, mathematics, creativity are linked in this activity.

The most delicate – and interesting for the researcher - phases of the activity are those in which we investigate on the capability of children to recognize in the elements of the project the corresponding ones of the ‘solid’ before cutting and folding. For this purpose we show them some pop-up projects representing, for example, rooms with the 'flat' drawings of a ward-robe, a chair, a bed, a table, a refrigerator, and some picture-cards representing a carpet, a picture, a cat, a window, a fruit-dish. We ask the pupils to put them in the project and then we verify together the correctness of their choices folding the project by 90°.

Primary school (level 2, pupils 8-10 years old)

Pupils work with the pop-up cube, the cuboid and composed solids. They make reflection on segments, angles, parallel, perpendicular and oblique lines and use instruments (goniometer, square, compass).

They begin by investigating on pop-up cube and comparing it with the 'true' one, grasping analogies and differences (pop-up solids are as *incomplete* as those words in which letters are half-erased and are *Gestalt* -recognizable *by reorganizing the field).* They discover that 2 faces out of 6, 8 vertices and 11 edges out of 12 are visible.

Pupils continue exploring the dynamic passage from the pop-up cube to the cuboid and analize as the changement of the dimensions of its edges influence the fold lines (the hill fold does not agree any more with the main fold of the sheet and this represents an anything but trivial logic obstacle to overcome).

The dynamic aspect of the pop-up technics enable to understand the cuboid is a particularization of prisms having a parallelogram as base, and the cube is an equilateral prism having a rhombic base.

The itinerary continues making pupils build solids composed by variously gathered together parallelepipeds. We have not worked with not right parallelepipeds yet.

The realization of a pop-up book can represent the goal of a didactic itinerary having as topic the discovery of the *principles* by which *the figures perform movements,that are consistent with the sizes of the book* and do not come out when it is closed.

Pupils discover these principles by solving experimentally opportunely organized concrete problems. Once more the errors are evident and it is possible to correct them by observation, reasoning, sensible use of mathematical knowledges, manipulation, enjoyment for discovering.

Conclusions

The pop-up instrument may effectively improve the capability to pass from tridimensional to level situations and vice versa. Being powerful on *logical* level and involving on *expressive* level, it is right both for being explored as a mathematical object (at growing levels of complexity in relation with the age of the pupils) and being utilized as an instrument in multi-disciplinary activities.

However we must underline that its quality can become its limit if the teacher loses sight of its logical potentiality. In this case, if the pupils do not control enough the conceptual aspects of the pop-up technics, they trend to lose sight of its bonds in their turn, to create the misconception that the object that they see realized (by the essential help of the teacher) is *really* that they have planned and to keep on working with a low attention to the *respect of the rules.*

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