The role of the teacher in promoting the pupils' interaction and metacognition in problem-solving activities. [1]

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***Abstract**: We present the partial results of a research, carried out with primary school teachers and a group of 80 pupils aged 8-10, aimed at modifying deeply the activity of problem solving. The pupils were asked to help an imaginary classmate having difficulties in solving verbal problems. We worked with a set of problems lacking of one or two pieces of information. Meaningful samples of the answers were discussed with the pupils in the classroom. All protocols were carefully analysed by the teachers and classified at metacognitive level (quality of the help given to Piera) and cognitive (way of solution). The activity gradually produced in the teachers a new awareness of the roles of problem solving in didactic and of their own specific role towards it.*

***Resumé:** On présente ici les résultats partiels d'une recherche, realisée par enseignants d'école primaire avec 80 écoliers de 8-10 ans, addressée à modifier profondément l'activité de problem solving. On a demandé aux écoliers d'aider une camarade de classe imaginaire (Piera) avec des difficultés à resoudre les problèmes verbales. On a travaillé avec un ensemble de problèmes qui manqueaient d'une ou deux informations. Dans les classes on a discuté plusieurs exemples significatifs avec les enfants. Tous les protocols ont eté analysés avec attention par les enseignants et classifiés soi au niveau métacognitif (qualité de l'aide donnée à Piera), soi cognitif (façon de resoudre). L'activité a peu à peu produit dans les enseignants une nouvelle conscience du rôle du problem solving dans la didactique et de leur propre rôle envers ce travail.*

1. Our research group has been working on problem-didactic in primary school (age 7-10) and middle school (11-14), as well as on the assessment of the pupils' competence in problem solving, for several years already.

The reason why we started dealing with this topic lies in the fact that the Italian history of Mathematics teaching has always privileged the *problem as an applied support to didactic*, whereas the *didactic of the problem ,* which is a standard in other nations' schools, is concretely absent. Many researches on the topic are appearing in Italy.

To start up an innovative didactic on problem solving in such a context means to foster activities that influence the diffused idea of the problem as an exercise of applied theory, in order to give it a more significant role within the syllabuses.

Of course the introduction of such activities implies the need of deep analysis by the teachers of the habits they have established in the classroom and of the assessment of the pupils' *attitude* besides their results.

2. There are many points of view under which such a complex question can be faced, and some of them can even become cognitive, metacognitive or psychological objectives. Our research stresses on the *linguistic* and *metacognitive* aspects, since it is our belief that mastering the language/s and the control of the process/es through its/their communication are

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the main factors of success in mathematics and, of course, in problem solving. Therefore verbal problems are one of the main themes of our research.

The activity was carried out during the school year 1996/97, lasted approximately 2 months and involved pupils aged 8 to 10.

The researchers, besides co-ordinating the activity, took part directly in the classwork. Actually, the aim was not only to study the pupils' behaviour in front of the solution of non-standard problems, but also to influence the teachers' methodology in order to raise a different problem didactic, the ability of leading classroom discussion on mathematical questions and the refinement of protocol assessment.

On facing a sequence of problem situations, the pupils had to substantiate their answers paying attention to their communicability besides their mathematical correctness. Therefore, especially as to metacognition, the following moments became important: oral and written verbalisation on OHP and collective discussion of the answers given.

As far as the teachers are concerned, besides the choice of the problems to be presented, an important methodological aspect was the increasing ability of the analysis of the protocols, according to a model that gives great importance to the study of the ways in which the pupils organise their answers.

3. The pupils were asked to help Piera, a girl of their own age, deal with some problems she couldn't solve (this situation created important and stimulating implications, also of social-affective nature). They were presented a sequence of nine problems lacking one ore more pieces of information; they didn't have to solve them instead of her, but rather explain her as clearly as possible why she hadn't been able to solve them.

The analysis of the protocols lead to classifying the answers into two levels:

(i) metacognitive (the *quality* of the help given to Piera);

(ii) problem-resolutory (the *correctness* and *clearness* of the procedures used). We would like to underline that the task didn't ask to actually solve the problem, but only to give Piera an explanation. Still, part of the pupils, for various reasons, decided to solve it anyway.

We shall see that the analysis of the protocols lead to finding out criteria for the evaluation of the answers, which suggested us short-term interventions (one-to-one or small-group conversations) as well as long-term ones (deeper workout). The classification of the protocols is reported in Appendix B.

In the following paragraphs we shall report excerpts of the diary of this activity[2].

4. The following text was presented to the pupils:

> Piera is trying to solve some problems, but she just can't manage it. See if you are able to help her by explaining her clearly why, in your opinion, she can't solve them. Here's the first problem:
> "Andrew buys a newspaper. The newsagent gives him 350 Liras change. How much was the paper?"

Actually we don't introduce the problem right away, we only show this text up to the words "she can't solve them", because we want to be sure that everybody understood the task "See if you are able to help her".

[2] Italic-printed paragraphs highlight our considerations on how the teacher's attitude influences the pupils' attitude and ideas towards PS.

This is not an easy stage, because the pupils are used to a standard context in which, usually, they must solve problems which are tightly connected to the theoretical topics they are being taught. Moreover, the didactic contract is usually influenced by stereotypes that have always emerged during classroom activities (collective or one-to-one discussions), and mainly while correcting protocols with the teachers, who recognise in most of the pupils' errors or ambiguities the result of their own input based on the equivalence: to solve problems = to do operations.

The most frequent justification concerns the teachers' difficulty in finding other ways of explaining to less skilled children how they should solve the problem. The dynamics playing can be schematised as follows:

(a) the teacher detects difficulties in the pupils;

(b) makes several attempts in order to foster the understanding of the logical aspects of problem solving;

(c) doesn't obtain meaningful results: on the contrary, the more the problems become complex, the more are the difficulties which make all efforts vain;

(d) defeated by not having been able to teach a method, they see no other alternative than arranging with there pupils a mechanical solution based on a series of "instructions" such as:

• "If there are two data, then there's only one operation to be done"; which implies "If the data are homogeneous (liras - liras, kilograms- kilograms and so on) almost certainly you must do an addiction or a subtraction, while if they're not homogeneous, then you must do a multiplication or a division";

• "If there is only one datum, you can't do any operation, so the problem cannot be solved" (of course this doesn't allow the pupils to recognise as data something that is not a number (e.g. 'half of' or 'triple as') or, as we shall see, to spot out any implicit datum (hidden in sentences like '...calculate how many kilometres they drive Monday to Saturday').

5. Once the task is explained, we complete the presentation of the text which globally constitutes the problematic situation the pupils must tackle (you will find here the analysis of only a few answers given to "the newsagent's problem"; the whole series of the problems that Piera couldn't solve is in APPENDIX A).

The pupils are invited not to ask any questions to the teachers and to write down their answer first into their notebook, then onto a transparency for the OHP, explaining it as clearly as possible (the task is not to solve the problem, but to help Piera; therefore the request is metacognitive instead of cognitive).

After that we collect all protocols and show some of them on the OHP, asking the children to express their opinion on whether they are clear and coherent to what was asked.

The discussion started by the comparison of the different protocols shows the pupils' attitudes towards many basic questions such as:

- what is a problem?
- what does it mean "to help" somebody having difficulties (when- very often - even the one who is supposed to help has his own difficulties and always needs help himself)?
- what is the teacher expecting from me when I get a problem to solve?

We are going to consider these aspects more in depth in the next paragraph, by analysing some prototype-situations occurred in the classrooms when we started a discussion on protocols we had purposely chosen and classified in advance.

6. We ask the pupils to express their opinions on answers of the following kind, which are quite usual:

• *"Dear Piera, you must read until you've understood everything".*

- *"Piera can't do it because she went to school and then went back home and forgot everything, so that's why she can't solve the problem".*
- *"Piera you can't solve the problem because you've been chatting while the teacher was explaining the task".*

The pupils consider factors which are not internal to Mathematics and resort to accusations (carelessness, lack of concentration), justifications (tiredness, excessive difficulty), advice (to read more, not to disturb) they always hear from grown-ups. The classroom discussion highlights the basic difference - at typically metacognitive level- between who use these explanations and the others, many of which underline their naive nature and their and unimportance as to the core of the meta-problem (explain Piera's inability). These protocols appear most frequently in 3rd and 4th grades and gradually disappear during the activity (from 32% to 1,7%). This experience shows that these are spontaneous reactions, basically very easily removable, which again, without this activity, would have stayed in the pupils' minds as stereotyped beliefs.

7. Another kind of answer, less frequent (about 8%) but more persistent (at the end of the activity still around 4-5%) is the following:

- *"Dear Piera you can't solve the problem because you have to do : data, drawing, operation and then answer".*
- *"I think Piera can't solve the problem because she can't do the operation ".*
- *"She must write down the numbers of the problem and explain what they mean. Then do question-mark and explain. You must do the graph and write the data you had put in there and then do another graph with the numbers and read the question.".*

These explanations are quite similar to the previous ones, still they are -so to speak- more 'internal' to mathematics.

They concern formal aspects of problem solving and actually reflect stereotypes induced by the teachers who in the end, wanting to give the weaker pupils the minimal tools for solving verbal problems, tie up in this net even the most skilled problem solves. Protocols of this kind also contain invitations to concreteness ("Piera, you must do a rough copy first") or to experimentation ("You must buy a paper and see how much it is and then you can solve the problem"). These texts declare the failure of stereotypes on showing the risk of trivialising problem solving activities and of building a dogmatic attitude right in those children who'd need to be stimulated in the opposite direction.

Unlike the previous ones, the mental attitudes underlying such problems are more difficult to be removed, since they represent steadily acquired beliefs the deconstruction of which creates a conflict with what they heard from the teacher thousand of times.

8. Another group of answers (frequency around 11%) is the following:

- *"I think you can't solve the problem because there's no price of the paper".*
- *"Piera can't do it because she didn't kwon how much the paper was".*
- *"Piera to me can't solve the problem because the text doesn't give the price of the paper".*

In these texts, the pupils mix up the missing datum with the question of the problem. To overcome such misunderstanding is definitely not easy, and discussion doesn't help significantly (it appears in average in 10% of the protocols, no matter the age of the pupils, and gets very little modified during the activity). It is closely tied to the logical structure of the problem and denotes a weak control of the meaning of the basic components of the problem situation.

9. Let us consider now another two groups of answers by comparing them (as we did in the classrooms in order to provoke discussion):

- *"Piera cannot solve the problem because the text doesn't say with how much money Andrew has paid the paper ".*
- *"Piera, to solve the problem you need to know how much money Andrew gave the newsagent. Without knowing it the problem is not solvable".*
- *"Piera, first of all you must find out how much Andrew gave the newsagent and then you must take off the 350 liras change and so you 've found the price of the paper".*

- *"Piera if you give him 1000 liras he gives you 350 back so you have to do the subtraction. Now continue!".*
- *"Piera you must do 1000 less 350 and the number that costs the paper comes out".*
- *"You must invent a number of how much the paper is? When you finish you can answer".*

The first group contains correct argumentation (obviously organised at different levels of clarity and linguistic competence) which are distributed this way (33% in third grades, 40% in fourth grades, 62% in fifth grades) and increase with time (from 21% on the first problem to 75% about the fifth).

The second group is rather less consistent in percentage (average 3,5%) and does not appear in third grades. The ideas contained are correct but - unlike the of the first group - show uncertainties at metacognitive level (to help Piera explaining her that *one datum is missing*) and the need of shifting to the cognitive one by inserting an invented datum or suggesting to do it.

The comparison of these two groups of protocols is interesting because the authors of the second one could fit in the Vygotskijan zone of proximal development, i.e. they express a essential understanding of the logical structure of the problem but have a partial metacognitive control on it. In fact, they offer a substantiated solution (inventing the value - not always *realistic* - of the missing datum) but don't help Piera concretely to understand why she cannot find it. The comparison with the protocols of the first group usually allows these pupils to understand clearly how to modify their attitude.

10. At the end of our itinerary, in order to test the level of learning, we present the pupils the following problem (the eight messages appearing in the text faithfully reproduce the main typologies of answers worked out by the pupils during the activity and can be compared with the items of APPENDIX B):

Dear friends, here I am again. How are you? I thank you for your help, but I need it again. The fact is that I didn't want to disturb you all the time, so I asked for help from the children of another school. They were very nice and sent me some messages among which I don't know what to choose, actually.

How shall I do? Please help me! Thanks a lot and bye bye!

Piera.

Here is the problem:

> *"Magda wants to share a box of 30 sweets in equal parts among her friends. How many sweets does each friend get?"*

And here's the messages. Does any of these really help me? Help me find it out!

a) Each friend gets 5 sweets.

b) Dear Piera you must do a division. Bye bye.

c) You cannot solve it because one datum is missing, actually how many sweets each if them get.

d) Piera you must pay attention when the teacher is explaining. Anyway the result is 6.

e) Piera you must write down all data, then do the drawing, do the operation and then you find the answer.

f) Piera, you cannot solve the problem because you don't know how many are Magda's friends.

g) I think you must multiply the 20 sweets by the number of friends.

h) Piera you just ought to divide 30 by 10 and you find 3 sweets each.

The pupils are asked different metacognitive performances. Namely, the pupil must:

(a) understand that one datum is missing, which one, and how to solve the problem;

(b) think upon the actual quality of the help given by each message;

(c) organise a hierarchy among the messages;

(d) substantiate his/her choice by writing.

All pupils show good control of level (a) and most of them find out - at least in the main streams - the hierarchy among the messages (b) (c). Substantiation (d), on the other hand, is very poor with no relevant difference (despite our expectations) between 3rd grades pupils and 5th grade ones. Therefore, discussion and reflection activities favour metacognitive performance rather than metalinguistic. Of course the age of the pupils has its role here, since they are more used to telling rather than describing thinking processes (metalinguistic operation). However, we have good reasons to believe that we have carried out - if correctly generalised - can start positive processes either in the pupils or in the teachers. In particular, as concerns the teachers, this experience made them aware of how shallow the usual didactic of problems is, of the way it limits the pupils' performance and of the fact that, consequently, this doesn't allow an effective assessment of their work.

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APPENDIX A

P1 Andrew buys a newspaper. The newsagents gives him 350 liras change. How much does the paper cost?

P2 Elisa buys a box of ice creams that costs 5000 liras. How much is each ice cream?

P3 All the steps in a ladder have the same height. The ladder is 20 metres long. How tall is each step?

P4A Meg takes 8 hairpins from a box. How many are left?

P4B Each hairpin in a box is made of 12 cm of metal thread. How much thread was used to make all the hairpins in the box?

P5 Our school has 2 buses. one of them drives 12 kilometres each day
Calculate how many kilometres the two buses drive Monday to Saturday.

P6A Mario buys some cans of cat food that costs 1100 Liras a can. Then he buys a toy for his kitten that costs 4500 liras. Calculate how much he spent altogether.

P6B Mario buys 7 cans of cat food and a toy for his kitten that costs 4500 liras. Calculate how much he spent altogether.

APPENDIX B

I Correct or virtually correct protocols

A Correct. Explanation organised at different levels of linguistic competence.

B Correct Vision of the structure of the problem, with the introduction of data chosen by he pupil (who has partial metacognitive control on the situation).

C Virtually correct.. Merely indicates the operation(s) to be done or contains a generic hint on the missing datum. Has metacognitive control on the task (to help Piera) but with stereotyped ideas (to solve a problem = to do operations) .

II Incorrect protocols:

D Not justified numerical solution (result of non-explicated operations with data chosen by the pupil)

E Confusion between missing datum and question of the problem.

F Protocols containing also calculations (classified separately; not included in this list because of too little room; will be presented orally).

G Simply as to the sequence of incorrect operations.

H Generic explanation focusing only on formal aspects of problem solving (data, drawing, operations, with or without invention of data).

I Generic explanation focusing on aspects external to mathematics (carelessness, tiredness, etc.) with or without suggestion of the result.

L Unconditioned surrender.