PRINCIPLES AND TOOLS FOR TEACHERS' EDUCATION AND THE ASSESSMENT OF THEIR PROFESSIONAL GROWTH

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We describe here the training model, developed within the ArAl project and characterized by the tight interrelation between contents to be taught (didactics of arithmetic and algebra in the perspective of early algebra) and teacher-educational processes, meant for teachers of the K-8 levels. We show how teachers, tutors and maths educators, by reflecting onto the Multi-commented Transcriptions (MTs), attain a shared development of the theoretical frame, of the methodologies and of the teaching materials that shall create the basis for the teachers' professional evolution. Finally, we tackle the question of how to assess teachers' professional growth by showing that MTs contain precious evaluation indicators.

INTRODUCTION

In the last years, many studies have been devoted to teachers' professional growth. Most of them underline that the teachers' change depends on the intertwining of different teachers 'inner' factors: mathematical knowledge for teaching, beliefs, emotions and awareness at different levels (Jaworski 2012, Mason 2008, Schoenfeld 2013). In particular, Sowder (2007) states that many of a teacher's core beliefs need to be challenged before change can occur. Schoenfeld stresses that the point is not what a teacher knows, believes or says, but rather how his/her knowledge and beliefs play out in the classroom. The change needed to traditional teachers to become non-traditional, inquiry-oriented teachers is not simple to achieve: it is very difficult to deeply affect the complex structures that constitute the basis for teachers' practices and strictly influence what they do, and their way of interpreting everything what happens. Our studies consider these aspects together: they aim at the renewal of the arithmetic and algebra teaching, they take into account both cultural and affective factors influencing teachers' action and develop the implementation of teachers' educational paths that are linked with the class activity and the students learning.

TEACHERS' EDUCATION: OUR APPROACH

As maths educators we are involved in activities for pre-service/temporary teachers or in-service teachers. In both cases, we develop studies on teachers' education with the aims: a) to validate our hypotheses about the effectiveness of our approach to favour changes in arithmetic/algebraic teaching; b) to individuate relationships between educational processes enacted and new teachers awareness, attitudes and behaviours; c) to confirm, extend and generalize results highlighting the essential conditions which determine them. Our research studies are essentially qualitative and develop through a methodology framed in the Italian research model for teaching innovations which is based on a sharp interaction with the teachers and a constant practice of shared reflections on the experimental paths jointly planned and enacted by the teachers about the mathematical content in play, the quality of their behavior, attitudes, knowledge, ways of thinking promoted in the students. Our interventions with/for the teachers are centered on the following key points.

Fostering teachers' reflection on their knowledge and on the coherence between their expressed beliefs and their teaching practice

The teachers involved in our educational programs (K-8 segment) have different cultural backgrounds, due to their different educational paths. Their beliefs about arithmetic and algebra are based on their knowledge but mainly on their emotions and beliefs, fruit of their previous experience (as student and teacher). these aspects constitute an often fragmented and fragile net of reference which underpin their teaching orientations. Most of the teachers believe that the teaching of arithmetic precedes the teaching of algebra; therefore, the procedural point of view prevails over the relational one, the manipulation of mathematical objects prevails over the reflection upon them, products prevail over processes, in problem solving the operational aspect prevails over the representation and interpretation leaving in shadow the control of meanings. As to the teachers' vision of their role in the classroom, the most widespread one is that of a ('soft') director, deriving from the belief that teaching means mainly conveying pieces of knowledge. Trying to make teachers overcome such beliefs through training means, first of all, leading them - through the *practice of the reflection* - towards a deep analysis of their knowledge and beliefs, which makes them call into question and gradually re-formulate their personal epistemology in a new frame oriented forward early algebra.

Developing open and redefined educational paths.

Teachers' educational programs often present a structure having a mono-directional motion within workgroups: mathematics educators limit themselves to giving teachers indications on how to improve their teaching, together with a set of tasks to be presented in the classrooms. In such a structure, teachers are basically users of a framework vision that they receive in top-down mode. On the contrary, our model suggests a multidirectional motion, characterized by a synergic network of relationships among teachers, tutors and mathematics educators, aimed at obtaining results that a single teacher would hardly achieve on his/her own. This synergy of relationships allows a continuous development of theory, methodologies and tools. We can therefore define this framework as being built in a bottom-up mode, within a tight interrelation between teaching contents and educational processes. A glossary shared with the teachers supports the educational process. It is a dynamic tool, to which teachers constantly refer. With the aim of constituting a real sharing of these theoretical elements, the glossary is progressively integrated according to the teachers' declared needs. Thus, a real 'formative communication' develops among teachers, tutors and maths educators in the inquiry community [1] (Cusi et al 2011).

Planning teachers educational paths and designing tools for data detection

The teachers addressed by our project are distributed in almost all Italian regions but they are subject to a frequent turn-over (funding decreasing, retirements, change of school directors and teachers etc.) and are fluctuating in duration. For this reason, the educational paths we develop for pre-service/temporary teachers are different from those developed for in-service teachers. In the first case more space is given to the study of classroom processes of *others* teachers with the aim of educating trainee-teachers to develop a good sensitiveness about constructive teaching (Malara e Navarra 2009, Cusi and Malara 2011). In the second case the educational paths are focused on the study of the teachers' *own* teaching processes and on the comparison between the attitudes and behaviors of the teachers involved in the same experimentations. This kind of activities are developed with the aim of making the teachers become aware of their own ways of acting and of the possible strategies to adopt in order to refine problematic didactical approaches. These activities also enable to deduce general results and to identify new research problems.

Collecting narratives and discursive data meaningful to teachers growth.

Our results are generated by different kinds of data: *notes about the meetings* during which the teachers and the researchers discuss the potentiality of the activities to be presented in the classes or, a posteriori, analyse the effectiveness of the teachers' ways of proposing the activities; *in-progress didactical materials* (tasks, students protocols, short excerpts of micro didactical situations, terms of the glossary, etc), selected by the teachers for the sharing of experimental results; *teachers interviews or discussions,* where aspects linked with their knowledge or methodological and emotional aspects are highlighted; *multi-commented transcripts* (MTs) [2] of audio (sometime video) recordings of classroom processes. Teachers' meta-reflections are encouraged, since they are asked to analyse the transcripts of class discussions commenting not only on students' interventions but also on their own interventions, trying to refer to the different constructs of the wide theoretical frame (see forward the point 'Methodology of Multi-commented Transcripts'). The collected data are studied to identify categories of teachers' behavior, characterized by set of actions modeling a metacognitive and effective teaching (Cusi et al. 2011).

THE CONTENTS: EARLY ALGEBRA AND THE ARAL PROJECT

Early algebra arises from the need to favour the pupils‘ construction of meanings in arithmetic in a pre-algebraic perspective across K-8 school levels. Its approach to arithmetic is based on specific principles: the anticipation of generational activities at the beginning of primary school; the social construction of knowledge, i.e. the shared construction of new meanings, negotiated on the basis of the cultural instruments available at the moment to both pupils and teacher; a focus on natural language as main didactical mediator for the slow construction of syntactic and semantic aspects of the algebraic language; identifying and making explicit algebraic relationships and structures within concepts and representations in arithmetic. In this context, our *ArAl Project: arithmetic pathways towards favouring pre-algebraic thinking* (Malara & Navarra, 2003) counters the traditional sequence of arithmetic/algebra teaching and suggests that their teaching is based on the intermingling between two disciplines, in the perspective of a continuity between primary and secondary school (see Figure 1).

	Arithmetics												
K	1	2	3	4	5	6	7	8	9	10	11	12	13
									Algebra				

Early algebra (EA)													
K	1	2	3	4	5	6	7	8	9	10	11	12	13
EA						EA-AT			AT				
						Algebraic Thinking (AT)							

Figure 1: Towards a new perspective of the teaching/learning of arithmetic & algebra

We claim that the main cognitive obstacles to the learning of algebra arise in unsuspected ways in arithmetic contexts and may impact on the development of mathematical thinking, mostly owing to the fact that many students have a weak conceptual control over the meanings of algebraic objects and processes, seen as translations of verbal sentences and in their mathematical status. Our hypothesis is that algebra should be taught as a new language from the beginning of primary school, so that one gets to master – through a set of shared social practices (collective discussion, verbalization, argumentation)– modalities that are analogous to the learning of a natural language: gradually appropriating its semantic aspects and putting them in their syntactic structure. For this reason, it is necessary to build up an experimental and continuously redefined environment, capable of informally stimulating the autonomous elaboration of a formal coding for verbal sentences by discussing them with the whole class. This process of construction/ interpretation/refinement of 'draft' formulas is what we call *algebraic babbling*.

This approach leads to a sort of Copernican revolution in the teachers' beliefs. It brings about awareness for the meaningfulness of their role, with reference to their position within the educational process. The teaching of arithmetic in an algebraic perspective is fostered by making teachers shift their attention from procedures towards relationships in arithmetic. Let us clear up this point by presenting four key-issues that exemplify our framework.

The equal sign and the duality process-product.

The usual reading of 5+6=11 is '5 plus 6 *is* 11': what's on the left to the equal sign is seen as an *operation*, whereas on the right it is seen a *result*. The two sides of the equal sign are interpreted as *ontologically different entities*. But the algebraic meaning is different: it indicates the equivalence between two representations of the same quantities, i.e. between two entities that are *ontologically equal*. We usually introduce this alternative perspective by discussing with teachers the words of an 8-year-old pupil: "It is correct to say that 5 plus 6 makes 11, but you cannot say that 11 'makes' 5 plus 6; so, it is better to say that 5 plus 6 'is equal' to 11, because in this case the other way round is also true."

Canonical and non-canonical representations of a number.

Similarly to what happens with the equal sign, writings such as $[(3+2)\times4]^2$ are seen as *operations* waiting for a *result*. In order to promote a reflection upon them in an algebraic perspective, we use the strategy of writing on the blackboard a list of facts concerning a specific pupil: name…, daughter of…, owner of a dog called…, and so on. We then explain that the situation is similar with numbers: each number can be represented in many different ways, through any odd equivalent expression. Among these representations, only one (e.g.: 12) is its *name* – the so-called *canonical* form, whereas the others (3×4, 3×(2+2), 48/4, …) are its *non-canonical* forms, each of which makes sense with reference to the context and the underlying process. This experience enables to understand that $[(3+2)\times4]^2$ is one of the many non canonical forms of the number 400. Being able to recognize and interpret these forms builds up the semantic basis for the understanding of algebraic expressions like -4p, ab, x^2y, k/3. The concepts of canonical/non-canonical form also allows to reflect upon the possible meanings associated with the equality sign; in $[(3+2)\times4]^2=400$ we don't see the operations and results anymore, but rather the equivalence between two representations (non- canonical and canonical) of the same quantity.

The duality 'Representing vs. solving'.

It is a widespread belief that solving a problem means identifying its *result*; this perspective focuses the attention on the *operations*. In order to bring about a change of perspective, it is necessary to move from the cognitive to the metacognitive level, where the solver *interprets the structure of the problem and represents it through algebraic language.* In the traditional perspective, solving a problem means separating the entities that are known from the entity to be found, then spot out the necessary operations. In the perspective of early algebra, the attention is concentrated on the (known or unknown) *entities*, and on the *relationships* among them. In a 'classical' problem for level 4, 'The sides of a rectangle are 3cm and 4cm long. find the perimeter', *the explicit entities are two* (3 and 4), the implicit one is the 'double' operator applied to each side and *the operations are two*; we obtain the writing (3+4)×2 which gives the result 14. But if the task is expressed as follows: '*Represent* in mathematical language the situation so that you can find the perimeter', *the entities are four*: the length of the two sides, the 'double' operator, the length of the perimeter (p), the *operations are two* but there is also a *relationship*: the equality between p and the representation of the process through which it is obtained. The sentence p= (3+4)×2 expresses all this is. This shift of perspective amplifies comprehension; in order to foster it, we use the principle '*first represent, then solve'*.

The duality transparent-opaque.

A representation in mathematical language is made of symbols that convey meanings, the comprehension of which depends both on the representation in itself and on the ability of those who interpret it. One could say that the canonical form of a number is poorer of meanings than its infinite possible non-canonical forms, for example: the tendency of immediately carrying out the calculation $5^2\times5^1\times5^3$ leads to a result, represented by its canonic form (15625), but the efficacy produced by the 'intermediate' representation 5^{2+1+3} gets lost, whereas it would allow to build the comprehension of why, in algebraic realm, $ab^2\times a^2b=a^3b^3$. We can therefore talk of a higher *opacity* for writings such as 15625, of a higher *transparency* for those like $5^2\times5^1\times 5^3$ and 5^{2+1+3}. Generally speaking, the transparency of the process favours the control of meanings, highlighting the underlying properties; it allows to understand possible errors and to clear up possible misconceptions which may arise.

THE EDUCATIONAL PROCESS: TEACHERS' PROFESSIONAL GROWTH IN THE PERSPECTIVE OF EARLY ALGEBRA

The Methodology of Multicommented Transcripts.

In order to fulfill these, and others, key-issues, we involve teachers in an activity of critical analysis, and consequent reflections, of the transcripts of audio and video-recordings of classroom processes, which we call Multi-commented Transcripts Methodology (MTM). It has the aim of highlighting the interrelation between the students' construction of knowledge and the teacher's behaviours in guiding them to perform this construction. The Multi-commented classroom Transcripts (MTs) are sent by teachers, together with their own comments, to e-tutors who make their own comments and send them back to the authors and other members of the team. The e-tutor highlight not only the positive aspects, but also the possible stereotypes, beliefs and behaviour that are often mistaken, and comment them with reference to the theoretical framework that is shared in the community of inquiry and, for some key-elements, also with the pupils. The joint reflection on the MTs strongly influences the development of theoretical, methodological, instrumental, material aspects (Units of the ArAl Series, papers, articles, learning objects) and supporting elements (website, blog, Facebook Group) aimed at offering teachers a cultural background that can help them act differently in the classroom (see Figure 2).

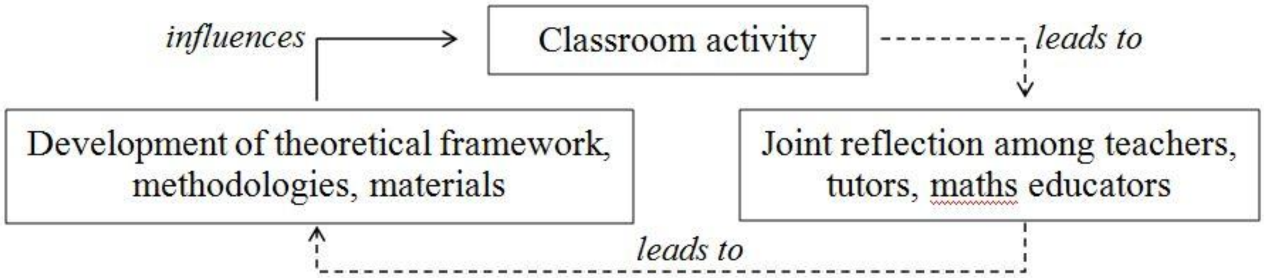


Figure 2: The cycle of teachers' mathematics education

HOW TO ASSESS

The problem of how to assess the teachers' change is an open research question. The strategy we adopt is to assess in an indirect and articulated way, through the MTs, the way in which the teacher's classroom action evolves during the training path. The factors we observe are: (a) the pupils' interventions; (b) the modalities in which the teacher interacts with the pupils; (c) the comments expressed by a teacher in the MTs and his/her reflections received in the sharing of his/her MTs within the community of inquiry. With this goal, the intersection of experience between both typologies of teachers (pre-service/temporary and in-service) becomes relevant in the strategies of the project, for example: meaningful excerpts from MTs produced by in-service teachers give vent to tasks for future teachers, aimed at assessing their ability to face specific classroom situations by hypothesizing contingent actions and foresee their development (Malara and Navarra, 2009).Some examples (teachers in service):

How pupils express themselves.

One can infer from pupils' sentences whether the teacher works in a pre-algebraic perspective, i.e. fostering the development of algebraic babbling and therefore inducing a 'metacognitive' attitude.

Episode 1(9 years old, grade 4)

Pupils are asked to represent in mathematical language the total number of sweets contained in six bags (each of which contains four chocolates and three candies).

Alessandro: I have written 7×6.

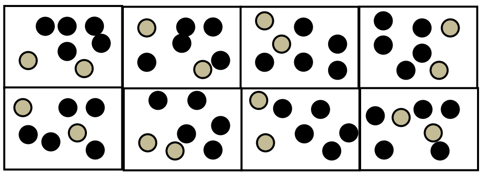
Miriam: I have written (3+4)×6: it is more *transparent*, Alessandro's writing is *opaque*. It means that it is not clear, whereas transparent means that you understand.

Miriam refers to the dichotomy *opaque/transparent* to express *how the non-canonical form of a number helps to illustrate the structure of a problematic situation*. This awareness in the pupil testifies to the fact that the teacher has acquired the ArAl theoretical framework and shares it with the pupils.

How teachers interact with pupils.

The way in which a teacher interacts with the pupils, and the role that he/she takes up have a strategic importance, because they influence the quality of the teaching.

Episode 2 (8 years old, grade 3)



The pupils have represented this situation in the mathematical language and reflect on the suggestions made by Alice n=5+2×8, Martina 5+2×8=n and Ada n=(2+5)×8.

Francesco: I think they are right because 5+2 represents the marbles that are in a box, by 8 which is the number of the boxes.

Maria: They are the same as Ada's but they don't have parentheses.

Teacher: Let's reflect on the presence of the parentheses. Do they change anything?

Andrea: I think they do, because 5+2×8 is equal to 21, while (2+5)×8 is equal to 56.

Bruno: It's true: the teacher said once that in a chain of operations you solve multiplications first.

Maria: This means they are not the same!

Francesco: That's right, the translation with the parentheses is the more correct one.

There are many elements here that let us assess positively, at different levels, the teacher's action: (a) *mathematically*: she has introduced the pupils to the use of letters, to the priority of operations within expressions, to the use of parentheses; (b) *linguistically*: she has fostered the organization of meaningful, complete sentences; (c) *metalinguistically*: she has promoted the reflection on the mathematical writings and their comparison; (d) *socially*: by inviting pupils into discussion, she has let them interact without her influence, listening to each other and having spontaneous dialogues; (e) *methodologically*: she has shared the theoretical framework with the class, by spreading words such as 'represent' and 'translate'. On the other hand, we make the teacher notice and reflect on the fact that (2) Francesco has referred '5+2' *to the marbles* and not *to their number*; (2) Andrea speaks of *result*, and pupils should be guided from the level of *calculations* to the level of *representations*.

The teacher's self-reflection and the suggestions offered by other comments.

The way in which a teacher interacts with the pupils, and the role that he/she takes up have a strategic importance, because they influence the quality of the teaching.
The comments written by the e-tutor and other experts – even in several diaries by the same teacher – allow to ascertain whether the classroom-leading strategies have changed (and *how*) during the training. Among the factors that influence the assessment there are, for example, the following issues: Does the teacher develop a wide range of roles in order to promote a refection onto mathematical processes or objects? Does he/she foster linguistic interactions by encouraging verbalization, argumentation, and collective discussion? Does he/she negotiate and share with the pupils the theoretical framework? Does he/she modify their initial points of view or does he/she seem unsensitive towards meaningful changes in his/her initial attitude?

Example 3 (12 years old, grade 7)

The class is tackling with the teacher the mathematization of a situation that requires the translation of the sentence 'for each 4 sage plants there are 6 rosemary plants'. The pupils attain first the formula s = (6/4)×r, then s =3/2×r. The teacher shifts to the level of interpretation and asks the pupils to determine the number of rosemary plants corresponding to 66 sage plants. Some pupils offer a solution by substituting the value 66 and s in the second formula. Then, another pupil speaks:

Mario: I have done it in a different way: 66:4=16 and the remainder is 2. Then I have done 16×6=96 because for 4 sage plants I have 6 rosemary plants. Then, since 4s=6r, I have divided by two that is 6:2=3 and after that I have added 3 to 96. The result is then 99.

Teacher: Bravo, now let's draw a graph by using the relationships that we know.

To this point, the teacher's reflection and the tutor's comment in the MT are:

Teacher: Here I should have lingered on what the pupil was trying to say, because I think that his reasoning is very interesting and it could have helped his classmates to see the same situation from different points of view.

Tutor: Ok. But before that, one should have underlined the improper use of letters as labels and the argumentation should have been interpreted at relational level, so as to show that his procedure is based on the non-canonical representation of 66 as 4×16+2, on recognizing the multiplicative relationship between 2 and 4 (2 is the half of 4) and on the implicit assumption of the distributive property. The pupil is referring to the non-simplified formula r=(6/4)s and transforms the calculation (6/4)×66 into (6/4)×(4×16+2)=(6/4)16+(6/4)2=6×16+3=96+3.

CONCLUSIONS

The question of teachers' evaluation is complex also because what we want to evaluate is a process which, in its nature, is continuously becoming and reflects daily changes of mood, energy, personal interest in the topic being dealt with, relationship with the pupils. All these factors interfere with the quality of the classroom activity management and therefore influence what one would like to evaluate: the effect of participating in the training on the strategies of leading a lesson. Our studies concentrate therefore on a formative evaluation of what arises in the MTs, a dynamic evaluation, which pays attention to nuances, apparently unimportant details, to micro-decisions that the teacher makes in just as many micro-situations, in which he/she often shows the co-existence of pre-existing beliefs and possibly hesitating opening attitudes, induced by the training. We hold this way promising, particularly if it is embedded in a educational context that constantly involves teachers and broadens the sharing of the MTs and of their reflections (along with their possible evaluation) to other actors of the community of inquiry, fostering a system of crossed evaluations.

These aims could be reached in a medium/long-term (two/three years or more) under these conditions: (i) enacting an educational project with several supporting tools; (ii) avoiding separation between theory and practice; (iii) building an environment in which effective circular relationship occurs between what happens in the classroom, the joint reflection of teachers-tutors-maths educators on classroom events, the shared effort of refining practices by relating it to the theoretical frame and developing materials, tools, as well as the assessment of the progressive change of the teaching through further classroom activity.

NOTES

1. We use the term 'community of inquiry' in the sense of Jaworski (2012). It involves mathematics educators (in the double role of didacticians and researchers), tutors (in the double role of mentors and teachers/researchers) and teachers.

2. The MTs are shared by e-mail and after critically analized during the periodical meetings of the ArAl community. Currently about 300 MTs have been put on the website < http://www.progettoaral.wordpress.com>. Together with other theoretical or practical materials, they constitute an integrated set of tools for teacher professional development".

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