Integrating teachers institutional and informal mathematics education: the case of 'Project ArAl' group in Facebook

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After a sketch of our ArAl project devoted to teaching/learning early algebra, we introduce our 'Progetto ArAl' group in Facebook, conceived not only to share and discuss among teachers didactical experiences, theoretical questions and materials but, more in general, to educate in informal way teachers in early algebra. For its features it can be said a non standard group (NSG) in Fb. The main question we put ourselves is: may a NSG become a community of practice*? To find an answer to this question we compared our group with a larger Italian group devoted to mathematics at primary school analyzing the interactions in the two groups launched by some common members. On the base of this comparison we delineate some hypotheses for the management of a NSG as a community of practice, where well known mentors and transparent theoretical guidelines allow the teachers consciously to approach the theory for the practice.*

Keywords: Early Algebra, Community of practice, Informal on line education, Teachers professional development.

Introduction

The ArAl project belongs to the stream of studies devoted to the renewal of the teaching in the arithmetic-algebraic area in the perspective of early algebra. It is characterized by the intertwining among: a) the activation in the classes of innovative didactical paths on early algebra; b) educational processes of teachers based on the critical analysis of the mathematical discussions developed inside the didactical paths. It promotes a relational approach to arithmetic of linguistic and metacognitive type, to be realized through socio-constructive modalities. The classroom activities are based on the negotiation of a didactical contract for the solution of problems according to the principle: "first represent, then solve". For room questions we cannot discuss deeply our theoretical frame (we refer to Cusi et al. 2011 and related references), here we simply recall some key aspects of it: (i) the plurality of representations of a given quantity, beyond the canonical decimal representation[1]; (ii) the identification and making explicit algebraic relationships and

[1] For instance, the number twelve, has a canonical representation in base 10, i.e. 12, but expressions such as 3×4, (2+2)×3, 36/3, 10+2, 3×2^2 are other ways to express the same quantity, we call them non-canonical representations of 12, each has its sense related to the process that characterizes it and offers pieces of information about the number. Being able to fluently shift among these forms allows pupils to easier recognize structural similarity among different numbers and to build the basis for understanding scriptures as a.b, -4p, x^2y, k/3).

structures underpinning concepts and representations in arithmetic[2]; (iii) the initiation to the essential algebraic cycle: *representing, transforming, interpreting* (Bell 1996) through *the devolution to the students* of*: a) the formalization of verbal relationships* individuated during explorative numerical activities (process named by us *algebraic babbling)*; (b) *the interpretation of simple algebraic sentences* both in themselves and with reference to a given context[3]; (v) the stress on natural language as didactical mediator in the slow construction of syntactic and semantic aspects of algebraic language[4].

Our work with and for teachers has always been realized in a community of practice, or better of inquiry in Jaworski sense[5], where the practice of the researchers and the one of the teachers meet, compare and develop in co-partnership and where, in addition to theory, methods and aims, values and expectations are shared (Cusi & Malara 2015). Because of the teaching in an early algebra perspective requires in the teachers, mainly the ones of primary school, a deep rebuilding of knowledge, beliefs, behaviours, and manners in the class, we have conceived specific modalities and apposite tools for teachers education. We simply recall here our *Multicommented Transcripts Methodology*, we have enacted to promote in teachers awareness of their own ways of being in the class and to guide them in managing mathematical discussions. Key tools of this methodology are the teacher's transcriptions of the classroom discussions enriched by written multiple comments (by tutors, maths educators and other teachers), *the MTs*. The joint reflections on each MT attain a shared development of the theoretical frame, of the classroom methodologies and of the teaching

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[2] For instance to see the equal sign, in writings such as 3+4=7, not only in its *procedural* sense of connection between an operation and a result, but in its *relational* sense, as an indicator of equivalence between *two different representations of the same number*).

[3] For instance, to recognize that the sentence 85=4×21+1 represents 85 through the quotient and the remainder of its division by 4 (or by 21), and in the same time - looking for the letter which stays at the 85° place in a sequence generated by the ABCD module – to recognize that the same sentence allows to understand what the letter is (the pupils have to interpret the term 4×21 as the part of the sequence done repeating 21 times the module and the remainder as the number of place of the letter in the successive module).

[4] For instance, two pupils express in natural language, and then translate in mathematical language, their different ways to calculate the number of pearls in the necklace ○○●●●○○●●●○○●●●○○●●●○○●●●○○●●●: the first pupil says "I counted white and black pearls and I added them" and translates: 2×6+3×6; the second says: "I saw that there are 6 groups, each group has 2 white and 3 black pearls and I multiplied 2 plus 3 by 6 and wrote (2+3)×6. The comparison of the two sentences allow the pupils to gain experience about the distributive property.

[5] We recall that a community of practice (CP) is constituted by a group of people who share a craft and/or a profession. The group can evolve because of the members' common interest in a particular domain or area, or it can be created specifically with the goal of gaining knowledge related to their field (Lave and Wenger, 1991). Jaworski (2003), referring to the joint work developed between maths educators and in service teachers about classroom teaching-learning processes, introduces the construct of Inquiry Community (IC) and underline that what distinguish a CI from a CP is that all the participants engage with inquiry as a tool to develop meta-knowing, a form of critical awareness that manifests itself in inquiry as a way of being.

materials that shall create the basis for the teachers' professional evolution. The productions of MTs became in the time a distinctive character of the Project Aral membership.

The ArAl Project Group in Facebook

Along the years many times we have been asked to make available ArAl materials to a greater number of teachers; for this in 2014 we opened the 'ArAl Project' group in Facebook. Fb is mainly used as a way to share experiences, practices and materials among teachers and other professionals (see for example Bodell & Hook 2011, Manca & Ranieri 2014), but in the last years it has also been used in educational activities for teachers (Staudt et al. 2013, Van Bommel & Liljekvist 2016). In our case, the initial idea was to spread themes and principles of early algebra among teachers and to motivate and help them in approaching it in the class but also to observe new, spontaneous didactical experiences, arising under the stimuli offered by the ArAl institutional courses. We believe that the Fb group can be a way to integrate institutional and informal education offering the teachers new occasions to promote their professional development. We started inviting expert teachers collaborating since long time in our project to become supporters of the group and to share their experiences with the teachers, recently involved in ArAl courses promoted by the schools, who have been invited to become followers of the Fb group. The fundamental methodological choices in managing the group are: our daily on line presence and prompt reactions to the teachers posts; the stimuli offered by the expert teachers posts through videos or pictures of classroom activities; our periodical posts about: mathematics questions and related theoretical references; examples of innovative activities, equipped by MTs, papers, powerpoint presentations for deepening the discussed questions and stimulating free experiments among the followers. The posts in Fb are classified in: 'like-agree' interventions; 'propositive-constructive interventions', doubtful-skeptical interventions; moreover meaningful sets of interventions related to interesting mathematical teaching questions are collected and commented in files put in our website. Periodical analysis of the data allowed us to highlight the interplay between our interventions and the teachers' ones, and to reflect on the teachers change. We discuss their evolution according to three temporal phases.

First phase (scholastic year 2014–15**).** In this first period, in front of a small group of teachers (in most part coordinators in the schools of the ArAl project activities), who were very active in posting documents related to their class activities as well as in commenting other posts, the other members were not so active, and their comments often were short and superficial. These teachers appeared *awed*: the most part of them had a feeble or null control over the early algebra topics and the strong difference among the competence of the expert teachers in the group and their knowledge in the arithmetic-algebra area did not encourage them to do more 'important' interventions. At the same time every day new members enrolled to the group. Some more expert teachers, members both in our group and in other groups for maths teaching, suggested us to visit them and in particular invited us to take part into the group 'Mathematics at primary school' (one of the most numerous and active Italian groups on maths teaching in the web, more than 5000 members), to offer our interventions whenever we seemed appropriate to do so. We call this last group a Standard Group (SG), in the sense that there are not pre-established leaders and that the exchange takes place freely through the sharing and negotiation of the individuals' knowledge. The comparison with the SG and other groups dealing with teaching issues brings in evidence that ArAl Project group is different from them, mainly for two reasons: (a) it deals with a well defined subject area, early algebra, it is structured according to a clear theoretical perspective for facing it, and it proposes methodologies, problematic situations, tools fitting with this framework; (b) it is daily supported by us and it is animated by experts teachers who may act as mediators among the members. Therefore we call it a non-Standard Group (NSG).

Second phase (scholastic year 2015-16): In this period we had continued to enter, as previously, examples of didactical activities, MTs, papers, powerpoint presentations but, at the same time - on our initiative or invited by the teachers - we had become more active in intervening on posts both in SG and in NSG. By way of example of this change in our strategy we focus on an episode: a post inserted by a teacher which received great attention (154 likers and 75 comments), started in the SG and developed, through reciprocal sharings, also in the NSG. The initial post contained a link to a note inserted in the Unit 12 of ArAl project and presented in the form of FAQ in www.progettoaral.it site. In this post it is developed a critical analysis of a typical Italian school practice, supported also by many textbooks, for introducing in primary school the decimal system of representation of the natural numbers: the indication of the units with the letter 'u', the tens with 'da', the hundreds with 'h' and so on (the so-called ‘marks'); thus there follow improper equalities such as 653=6h+5da+3u. Because of the impasse generated in the SG, a follower - a member of both groups - asked us for an intervention on this topic. In a comment of a theoretical and linguistic type Malara wrote:

“The symbols h, da, u represent words of Italian language. They are categorical terms that refer to orders of magnitude and they are used as ‘indicators of quantity’. They are useful for bringing the pupils to shift from the experience with the abacus - where an assigned quantity is split into opportune multiple of powers of 10, operating for successive groupings of 10 – to the representation of the result of this operation through a string of symbols, each between 0 and 9 (extremes included), from which the name of the given quantity was born. This means that, for example the string 6h, 5da, 3u synthetizes the verbal sentence ‘the quantity is constituted by six hundreds, 5 tens and 3 units’ which generates the name of the number 653. The translation into the arithmetical language of this verbal sentence requires the conversion of the term ‘hundreds’ in the arithmetic operator ‘×100’, the term ‘tens’ in ‘×10’ and the ‘units’ in ‘×1’ and the conversion of the connector ‘and’ in the operation of addition ‘+’. So, the total verbal sentence is translated into ‘3×100+5×10+3×1’. The sentence 653=6h+5da+3u is improper because it mixes the two languages, verbal and arithmetic, and confuses the metacognitive plan with the operational one”.

While the debate on this issue was developing in the GS, many teachers did not understand why in the ArAl project sentences as 653=60+50+3, 653=6×100+5×10+3×1, $653=6\times10^2+5\times10^1+3\times10^0$ were proposed as correct and not the one they used, and opposed resistance to accept the explanation that 653=6h+5da+3u is to be discarded because it is not a correct representation in mathematical language. To facilitate this understanding, the improper mingling between verbal and arithmetic languages has been pointed also using examples of verbal sentences with words in two languages; the discussion then focused on the correct and incorrect representations of a natural number, the concept of ‘equality’ and on the meanings of the symbol ‘=’.

Third phase (June 2016 to now) The analysis of the dynamics arisen and the kind of the comments posted in NSG and SG led us to the identification of some thematic questions who have given us valuable indications on a question that we did with increasing frequency: may a group with the characteristics of ArAl group become a *community of practice*? If the answer is yes, in which ways may this happen? How may a gradual constitution of a library of shared knowledge be put in place? This leads us to identify some answers to these questions concerning the prevailing attitudes of teachers who enroll in these groups. We discuss them articulating in the following points.

Features of a SG and of a NSG.

Members of a SG feel all *equal*: they exchange information, questions, requests without demanding to receive in-depth and substantive answers; they hope to share with their peers working suggestions which are at the level of their knowledge and of their willingness to get involved. Individual growth stems from the strength of exchanges and the wealth of experiences put into circulation. Internal leaders emerge, who often are recognizable more by the diligence than by the quality of interventions; they often are the most convincing not for their knowledge but because they expose themselves more than others, writing frequently comments. Members may find appealing ideas for new activities but their enthusiasm is not supported by an adequate knowledge; they express insecurity when discussing their colleagues' proposals of those embryos of new ideas. Everyone feels free to comment on impulse. On the contrary, a NSG as 'Progetto ArAl' gives the majority of subscribers some (cultural and psychological) constraints that limit them in exposing their contributions. The same dynamics occur in a working-group in which an expert is present. But then: if it is understandable, for the reasons explained, that a GS exceeds 5000 subscribers, how has to be interpreted the success of our NSG that in two years is approaching 1000 members? The answer could be given with a metaphor: the members *have the impression of living a moment of institutional training*. They know that in the NSG there are experts involved in the discussions, extemporary comments should be avoided and the participants are invited to put forth questions and to interact with others. At the same time they know that there are not 'free rounds' (as often happens in the SGs, where a rich variety of cues are offered but they often remain at a messy, unspecified, superficial level) and should deal with the theoretical aspects through an *individual study*. In fact, at the base of ArAl Project there is an organic vision that aims to propose a framework on early algebra, offering the participants opportunities to reflect on knowledge, beliefs, stereotypes. They accept a commitment which attracts them: to avoid free, trivial conversations or Pindaric flights.

How can personal experiences, beliefs, inclinations be influenced by interventions based on strong theoretical references?

The interventions on SG highlight different objectives between mathematics educators and teachers: basically, specialists focus their interest on the discipline, the teachers on their pupils. These different perspectives can create misunderstandings or misinterpretations. Then, in the NSG, mediations between them are necessary, that is: on one side the founding principles of mathematical knowledge – in our case of early algebra - have to be respected, but on the other side, at the same time, it has to be offered to the teachers a certain 'serenity' about the fact that deepenings and changes of perspectives in teaching do not affect learning, but on the contrary pave the way for subsequent extensions of mathematical concepts. There is a strongly felt concern that pupils do not understand or that a concept is too difficult or inappropriate (of course this concern is correct because teachers have the responsibility of the learning of their students, so they constantly consider the difficulty and feasibility of new proposals).

Limited capacity to distinguish between different types of knowledge.

The posts and comments put in evidence that most teachers, along the years, reach their convictions grounding them more on the accumulation of heterogeneous strategies, methods, tools than on their consistency. One of the consequences of this behavior is that teachers confront themselves superficially with the theoretical references. For example:

> Elena: I think sometimes that famous ‘didactic contract’, of which we all partake the negative effects on pupils, has been moved up on teachers: “It is so, Tom said, Dick reiterated it”; someone makes it [i.e. the didactical contract] arguing and expressing his/her opinion (experience counts, anyway!); someone else makes it ‘getting on the chair'. But: be they teachers or pupils or propagandists or colleagues, always 'didactic contract' is.

To what is Elena referring when she writes “the negative effects on pupils”? Her so peremptory statement was not reconsidered in the later comments: what does this mean? It could indicate that it has not been understood, or that it has been read superficially, or that it is not shared, or that it is an unfamiliar concept and no colleague wants to explore it. This short episode shows indeed that there are interactions between members, but in general they go on without reaching a real conclusion; at most, members achieve a superficial agreement, or a generic praise, or they remain on their positions. It would have been important to ask Elena what she means with this term (originally it is a theoretical construct by Guy Brousseau). Probably such statements would not have been made in our NSG. This might be a limit for the group because many convictions would not be expressed for a kind of compliance towards the coordinators experts. A low understanding of the key aspects of mathematics education (at the primary-secondary school level) favours the choice of cues - references, materials, paths, methods - that fit with the convictions and the personality of the teachers more than with the organicity of the knowledge taught. In this way, those facilitators that favour the perspective of *making* are privileged. The weak capacity to connect effectvely the suggestions of experts and mentors implies that one prefers a ‘do-it-by yourself’ shared with those who are felt as fellow-travellers: if an activity, a text, a method are exalted or defended by other members of the group, they may be adopted, or at least tested. Often, the length of an experiments is short because the activities are heterogeneous, have ‘little oxygen’ (the interest on them goes out early); almost immediately they are put aside without any reflection in general terms, mostly on the basis of local success achieved by pupils (or, more trivially, because they appear ‘nice’).

New characters emerging in NSG

The dense interactions developed in the NSG together with the offered theoretical and practical supports brought some new attitudes and awarenesses in the members of NSG. The members begin to understand that a new approach to the arithmetic and algebra teaching lies on a different role of the teacher. As to this a decisive importance assumes what J. Mason has called *the art of noticing* the classroom micro-situations for being ready to adopt the opportune micro-decisions (Mason

2002), intertwined with the attention to the *languages* and to the continuous recourse to the *argumentation*. Thanks to our frequent interventions where we underline that: a) a math teacher has to control a *plurality* of languages and that also a formal language must be monitored at two levels, the *semantic* one about the meanings and the *syntactic* one about the structure of the sentences into play; b) the weak control over grammatical/syntactical aspects of a sentence in mathematical language leads to temporary and unstable jargons in which the meanings assigned to the symbols are dictated by an apparent common sense that reduces the difficulties, promotes an immediate but feable understanding that leaves the problem unsolved; we observed in the activities posted by many members a bigger attentions forwards the translations questions between verbal and fromal languages and the increasing use of argumentations in their students. From a methodological point of view, thaks to our suggestions,when the teachers publish at the NSG the post of an activity, they begin to understand that it is not enough to insert some captivanting images, but that it is necessary to equip them with a presentation that synthetically shows the activated competencies and that includes the most meaningful protocols, the path in which the activity is inserted, how it develops in the next steps, the theoretical references (ArAl Units, items on the website, Powerpoint presentations, papers). Our idea is to slowly bring them to approach the MTs methodology. An important contribution in this sense is offered by an increasing number of members the NSG, who are not involved in ArAl experimentations but following the project in a convincing way (teachers educators, mentors, collaborators of publishing houses, members of other research groups). Thanks to this people the posted comments begin to be richer and meaningful; the authors express their ideas also asking for experts' suggestions aimed at promoting new and more adequate behaviours for teaching arithmetic/algebra in a *relational* perspective. So, posts and comments begin to produce virtuous relations which gradually enhance the system: the posts induce comments of increasing quality, which generate important feedbacks in the organization of the succesive posts.

The recent mutations observed in the NSG members' posts delineates a new character of their participation which appears in tune with our aim to build a shared identity in the NSG and effective in offering contributions which can bring it to become *a community of di practice.* As to this, particularly meaningful appear the recent initiatives generated by the NSG discussions concerning the publication in the ArAl project website (http://www.progettoaral.it/) of two documents, respectively devoted to: (1) the most interesting classroom episodes presented by the NSG members, with the main related comments; (2); the early algebra papers written by members external to the project and inspired by our previous productions. Next to this we have to consider the request expressed on the web by several members of NSG to organize some ArAl meetings of one or few days to allow the participant know themselves *de visu* and to plan some common work. It seem us that these new tends in NSG may generate inside the group, mainly with the more sensitive and expert members, an embryo of a *community of inquiry.* In this frame institutional and informal ArAl educational initiatives are developing important merging points.

Final considerations

A NSG as 'Progetto ArAl' may initially disorient new participants, but its own structure can be considered its force because many of them declare that they appreciate the possibility to join to a group where experts favor an organization of knowledge according to transparent and shared principles. On the base of the observations made, we formulate some key points related to early algebra for the management of the NSG so that it can become a significant community of practice in this field: (a) to help teachers understand not only merits and limitations of instruments and didactical strategies that they implement along the years, but above all the importance of their coherence and adherence to a set of theoretical principles, such as: the importance of languages and, consequently, of the translation between them; (b) to bring teachers to consider the perspective of the generalization since the first years of primary school, highlighting the structural analogy between representations of the various occurrences of a phenomenon and guiding their modeling; (c) to propose any time, during the discussion on the issues raised by the members, gradual general frameworks, accompanying them with clarifications, insights, extensions which give answers for doubts, perplexities, conflicts emerging from the discussion. The basic idea is that the theory should be gained through a gradual process of refinement of knowledge in a continuous exchange among the members of the group, adapting explanations and deepenings to the difficulties or to the resistances and injecting now and then proposals of mini-workshops.

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